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Survey on lateral buckling experiments, Lehigh University, (1960)

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WELDED CONTINUOUS FRAMES AND THEIR COMPONENTS

SURVEY OF LATERAL BUCKLING EXPERIMENTS

by

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I. INTRODUCTION

The original purpose of this report was to present a survey of lateral buckling experiments, in order to (1) study what has been done in the past, (2) collect all test results and methods of testing to facilitate further experimental research of beams in the post-buckling range, and, (3) summarize systematically the test data and the proposed design suggestions in order to make a set of unified data available to structural engineers.

The first of these purposes was achieved in a literature survey report* on lateral buckling published in August 1960. In July 1960 another paper** was published which gave a thorough summary of all available test data and design recommendations. Thus the third purpose of this research was also accomplished. The second purpose, however, was only done partially. A recent dissertation*** and a

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* G.C.Lee, A SURVEY OF LITERATURE ON THE LATERAL INSTABILITY OF BEAMS, WRC Bulletin No. 63, August 1960.

** J.W.Clark, and H. N. Hill, LATERAL BUCKLING OF BEAMS, Proceedings of the ASCE, Vol. 86 (ST7) (July 1960).

*** G.C.Lee, INELASTIC LATERAL INSTABILITY OF BEAMS AND THEIR BRACING REQUIREMENTS, Ph.D. Dissertation, Lehigh University, 1960.

progress report* have discussed the methods of testing and presented a new method of testing beams in the post-buckling range.

The test results of all lateral buckling experiments will be presented in a systematic manner in this report. The sources of the data are taken from the references listed in the end of the report.

A summary of the collected data is given in Table I. The test results are plotted in the figures following Table I. The sequence of presentation is arranged according to the following criteria:

1) Shape of Cross Section
   - a) Double-Symmetrical
   - b) Mono-Symmetrical
   - c) Unsymmetrical

2) Material
   - a) Steel
   - b) Aluminum
   - c) Xylonite

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* G.C.Lee and T.V.Galambos, THE POST-BUCKLING STRENGTH OF WIDE-FLANGE BEAMS, Fritz Laboratory Report No. 205E.12, June 1961.

3) End Moment Ratio

   a) Unity

   b) 0.5, 0, -0.5

   c) Minus Unity

4) Range of Buckling

   a) Elastic

   b) Inelastic

2. DESCRIPTION OF FIGURES

Fig. 1 All test results which are plotted in this figure are for structural steel WF sections subjected to constant moment. Equal loads were applied at the quarter points. In five of the series, one of which was in the inelastic region, the beam ends were simply supported while in the others some end restraint was present due to the type of connection (top and seat angles in one case and web angles in the other case). The parameters used in plotting results are M_{max}/M_p and Ld/bt.

The test results are compared with an approximate equation for critical stress (solid line) in which this stress is appropriately converted to M_{max}/M_p. The dotted line represents the familiar AISC design formula $f_{cr} = \frac{12000}{Ld/bt}$; the stress is similarly converted to M_{max}/M_p.

It can be seen that the specimens with partially restrained ends have a higher maximum moment than those with no end restraint.

Fig. 2 These are results from two series of tests of aluminum wide-flange sections subjected to constant moment. The ordinates and abscissas are M_{max}/M_p and Ld/bt respectively. It can be seen from the graph that the beams with end restraint are somewhat stronger.

Fig. 3 In this figure results from tests considering the effect of moment gradient are considered. The moment gradient in each case was equal to zero (i.e. $\rho = 0$) however the end restraints were different as can be seen from the three diagrams on the graph. The beam specimens were I shapes made of aluminum.

In the first series the ends were simply supported and subjected to a bending moment at one end. The beams in the second series were also simply supported but subjected to equal and opposite moments at their ends. Only one half of the span was considered for the zero moment gradient. It is readily seen the half of the span which is not considered offer some restraint to the half that is. In the third series cantilever beams with loads at their free end were tested.

Though there was no information as to σ_y for the third series, 30 ksi was assumed to allow for the non-dimensional parameter of M_{max}/M_p to be used for all three series.

Fig. 4 Tests were made on simply supported beams with concentrated load at their centers and the results are plotted in this figure. The sections tested, which were made of light alloy, were I shaped. Since there was no data available for thicknesses and σ_y's of the specimen M_{cr}/bd^2 and L/b were chosen as parameters.

Fig. 5 In this figure are plotted results from tests which are similar to those represented in Figure 4. The cross sections were I shaped as before but the material was xylonite. The same parameters, M_{cr}/bd^2 and L/b, were used.

Fig. 6 This figure represents results from tests on aluminum, I shaped beams with various moment conditions ranging from constant moment to a moment gradient of -0.5 (see diagrams on graph). The parameters chosen were M/M_p and Ld/bt.

Fig. 7 The loading conditions for tests represented in this figure were the same as those in Figures 4 and 5. In these two series however, the beam sections were channels. The material was aluminum. The same parameters were chosen as in Figures 4 and 5 and for the same reasons.

Fig. 8 The data plotted in this figure is from tests on simply supported beams with concentrated loads at the center. Four different cross-sectional shapes of xylonite were tested as is shown on the graph. It should be noted that each of the sections tested had approximately the same area. The parameters used were M_{cr}/Z and L/b where b is defined for each shape on the graph.

Fig. 9 This figure shows the results from tests on beams with unequal flange areas subjected to equal end moments. The material was again xylonite. The parameters chosen were M_{cr}/Z and L/b where b is taken as the width of the wider flange.

Fig. 10 Two different sections were subjected to constant moment and the results are compared in Figure

10. For the L/b's of 16 and 36 the σ_y for the material was 39,800 psi and for L/b's of 24 and 52.5 the σ_y was 44,900 psi. The cross sections were ⊏'s and Z's and made of aluminum. The parameters used were M/M_p and L/b.

Fig. 11 Various lengths of I shaped sections of aluminum were examined under constant moment and the results of the tests appear in Figure 11. In two of the tests the narrow flange was in compression and in the other two the wide-flange was in compression. The parameters used were M_{cr}/Z and L/b where b is the width of the compression flange.

Fig. 12 In the series represented in this figure two loads having a certain ratio for each test were placed at the quarter points of a simple span. The shapes tested were I's made of xylonite. The ordinates and abscissas are n_1 and n_2 respectively.

$$n_1 = \frac{Pc_1}{(Pc)_{single}} \quad \text{and} \quad n_2 = \frac{Pc_2}{(Pc)_{single}}$$

where $(Pc)_{single}$ is the critical load when it is applied alone at one of the quarter points. Pc_1 and Pc_2 are the critical loads when they are on the span simultaneously in a certain ratio.

Fig. 13 This figure represents the results of a series of tests which considered the effect of position of load on a single span. The material was light alloy and the sections were I shaped. The load was placed at a certain position αL from one end. The parameters chosen were α , which varies with the test, and $(Pc)\alpha/(Pc)$ center.

Fig. 14 In this figure is presented the only available data on inelastic buckling. The specimens tested were steel wide-flange sections. Six different sections were tested as simple spans loaded at the center for various L/r_y ratios. Five sections were tested with equal loads symmetrically placed on a simple span. Finally four more were tested with unequal loads symmetrically placed. The parameters used to plot the points were M_{max}/M_p and L/r_y where L is defined on the graph.

3. NOMENCLATURE

M_{max}	= Maximum moment attained
M_p	= Plastic moment = $Z\sigma_y$
M_{cr}	= Moment at buckling
L	= Length (as defined on figures)
d	= Depth of section
b	= Width of flange
t	= Thickness of flange
ρ	= Ratio of smaller moment to larger moment in one span
Z	= Plastic modulus
σ_y	= Yield stress
P	= Load
f_{cr}	= Critical stress
f	= Shape factor
M_o	= Applied moment or maximum bending moment across span
r_y	= Radius of gyration about y-y axis
Pc	= Critical buckling load

4. ACKNOWLEDGEMENTS

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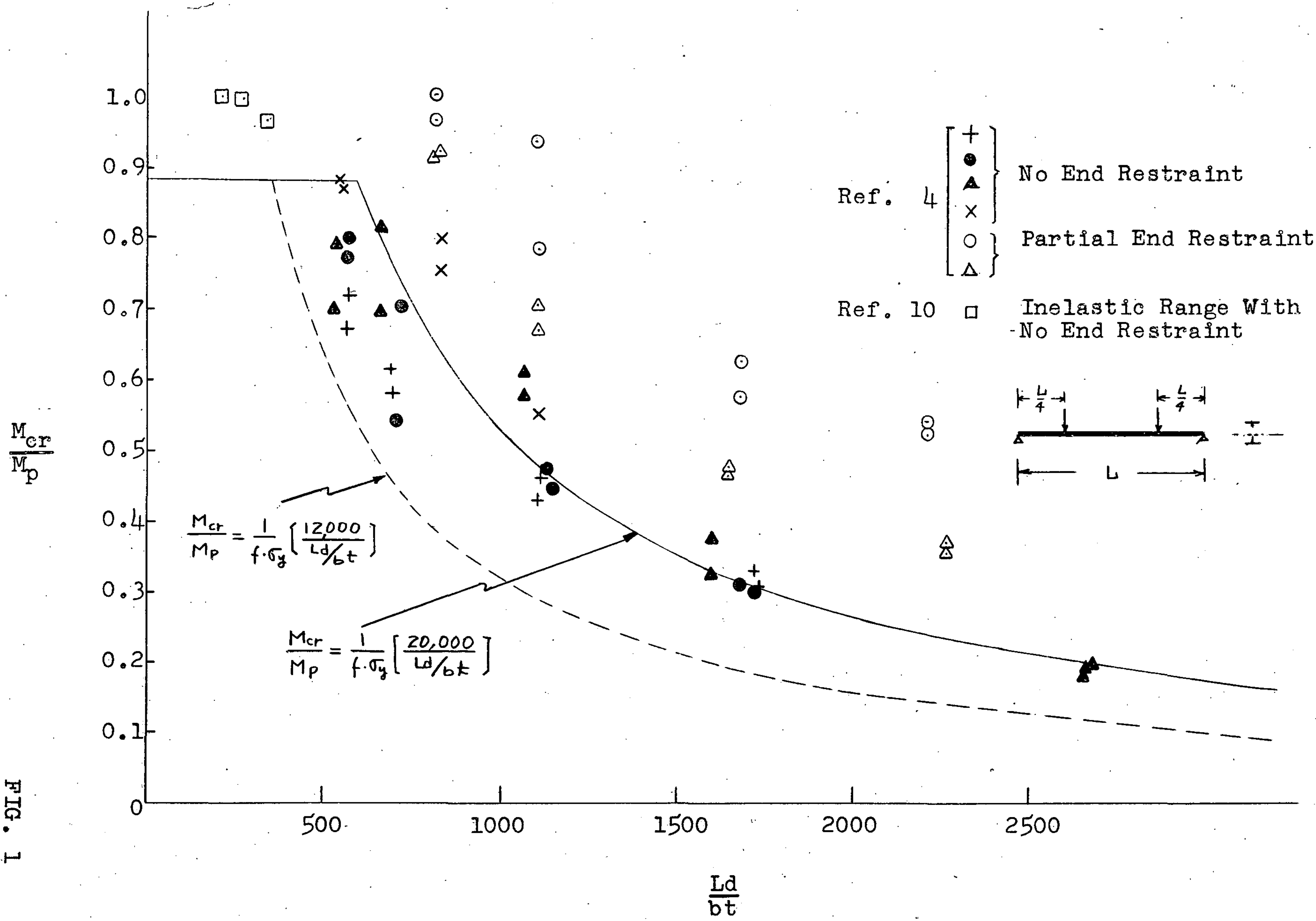
5. TABLES AND FIGURES

TABLE I

TABULATION OF FIGURES

FIG. No.	ABSCISSA	ORDINATE	MATERIAL	MOMENT CONDITION	SHAPE OF SECTIONS	REFERENCE NUMBER	RANGE
1	Ld/bt	M_{cr}/M_p	Steel			4,10	Elastic
2	Ld/bt	M_{max}/M_p	Aluminum			1,13	Elastic
3	Ld/bt	M_{max}/M_p	Aluminum			1,2	Elastic
4	L/b	M_{cr}/bd^2	Light Alloy			2,3	Elastic
5	L/b	M_{cr}/bd^2	Xylonite			2	Elastic
6	Ld/bt	M/M_p	Aluminum			1	Elastic
7	L/b	M_{cr}/bd^2	Aluminum			3	Elastic
8	L/b	M_{cr}/Z	Xylonite			2	Elastic
9	L/b	M_{cr}/Z	Xylonite			7	Elastic
10	L/b	M/M_p	Aluminum			8	Elastic
11	L/b	M_{cr}/Z	Aluminum			12	Elastic
12	n_2	n_1	Xylonite			2	Elastic
13	α	(Pc) /(Pc)	Light Alloy			2	Elastic
14	L/r_y	M_{max}/M_p	Steel		WF	10,13,18	Inelastic



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
500
1000
1500
2000
2500
$\frac{M_{cr}}{M_p}$
$\frac{Ld}{bt}$
Ref. 4
No End Restraint
Partial End Restraint
Ref. 10
Inelastic Range With No End Restraint
$\frac{M_{cr}}{M_p} = \frac{1}{f \cdot \sigma_y}\left[\frac{12{,}000}{Ld/bt}\right]$
$\frac{M_{cr}}{M_p} = \frac{1}{f \cdot \sigma_y}\left[\frac{20{,}000}{Ld/bt}\right]$
L/4
L/4
L

FIG. 1

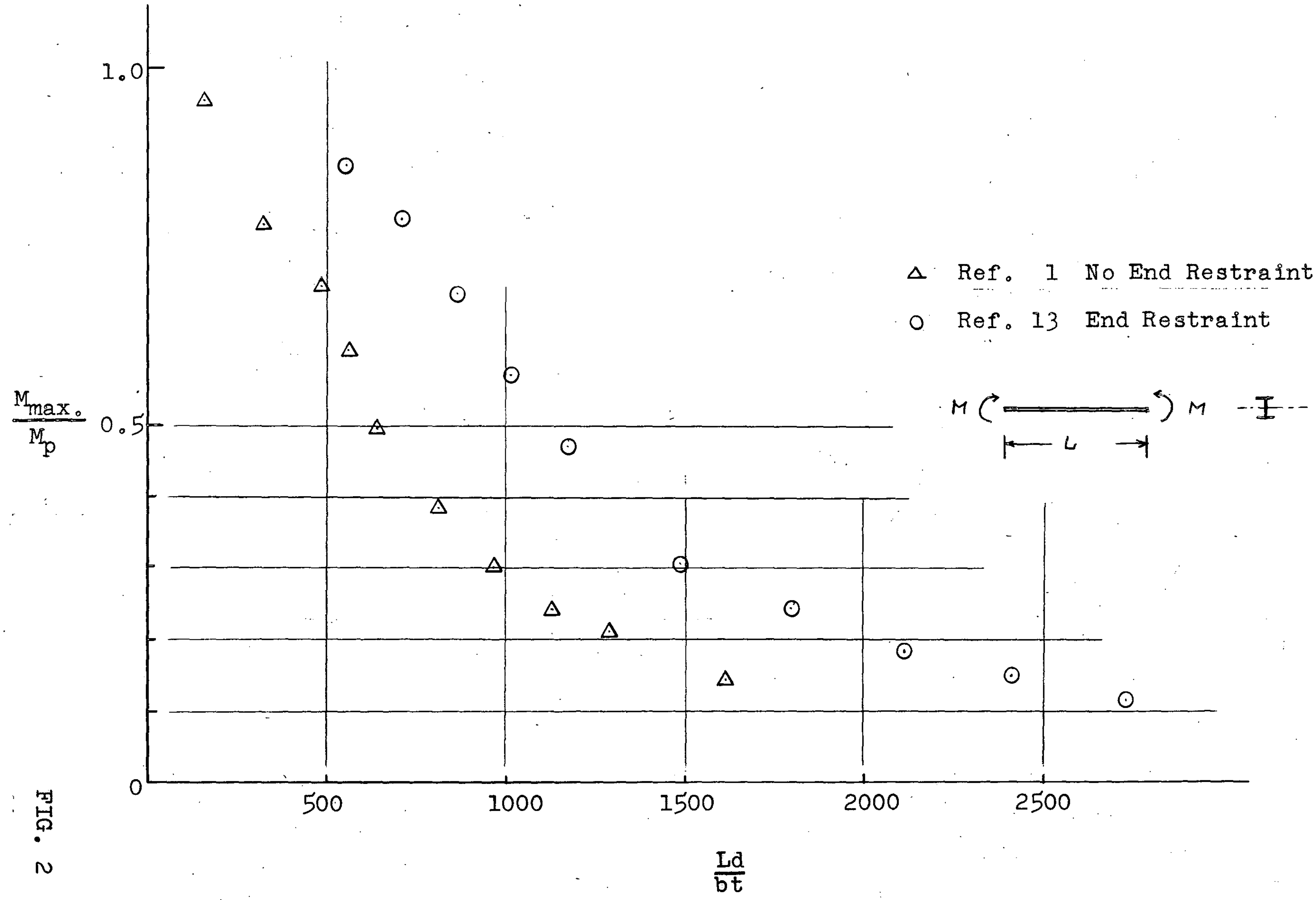

1.0
0.5
0
$\frac{M_{max.}}{M_p}$
500
1000
1500
2000
2500
$\frac{Ld}{bt}$
△ Ref. 1 No End Restraint
○ Ref. 13 End Restraint
M
M
L

FIG. 2

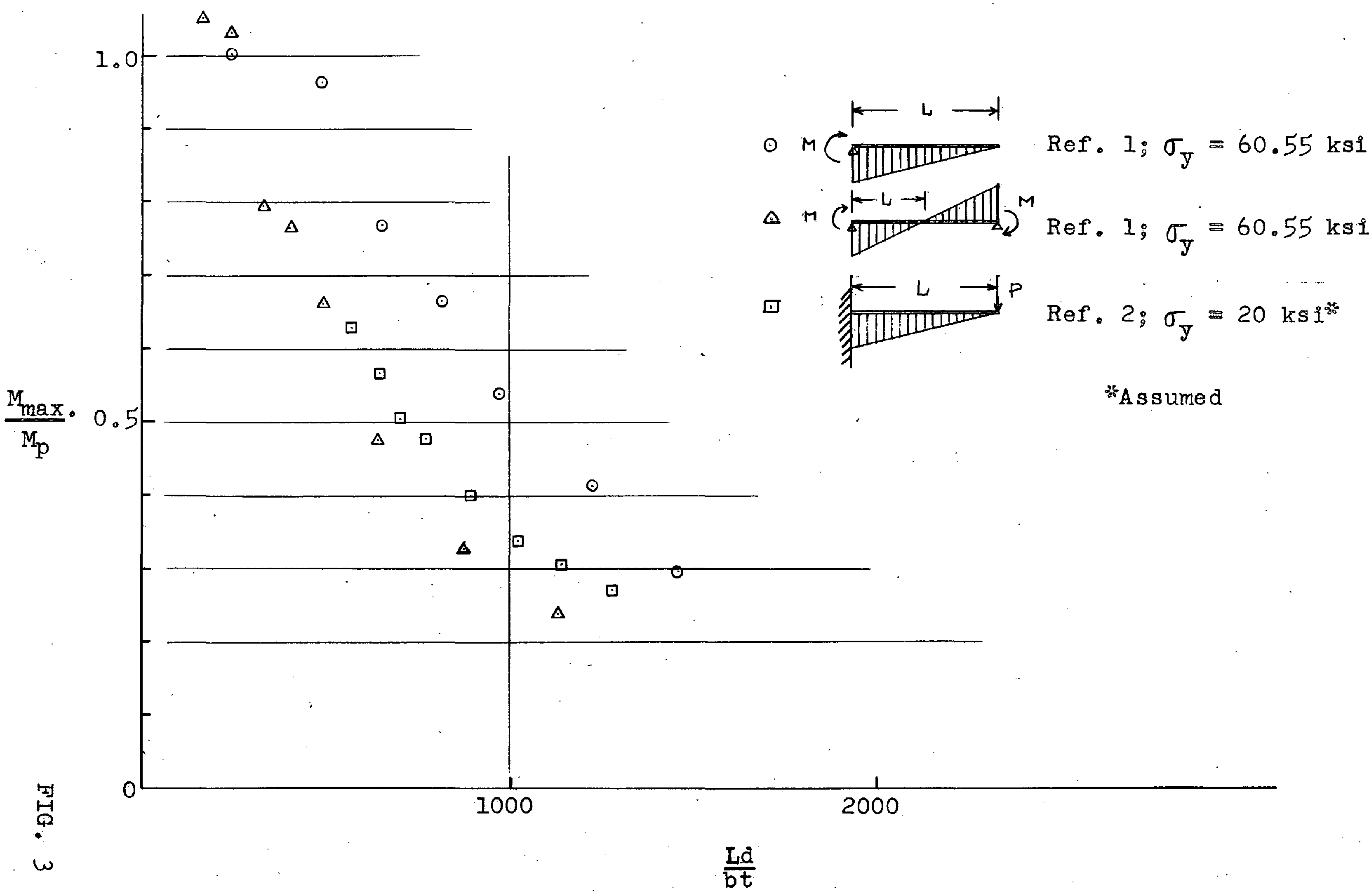
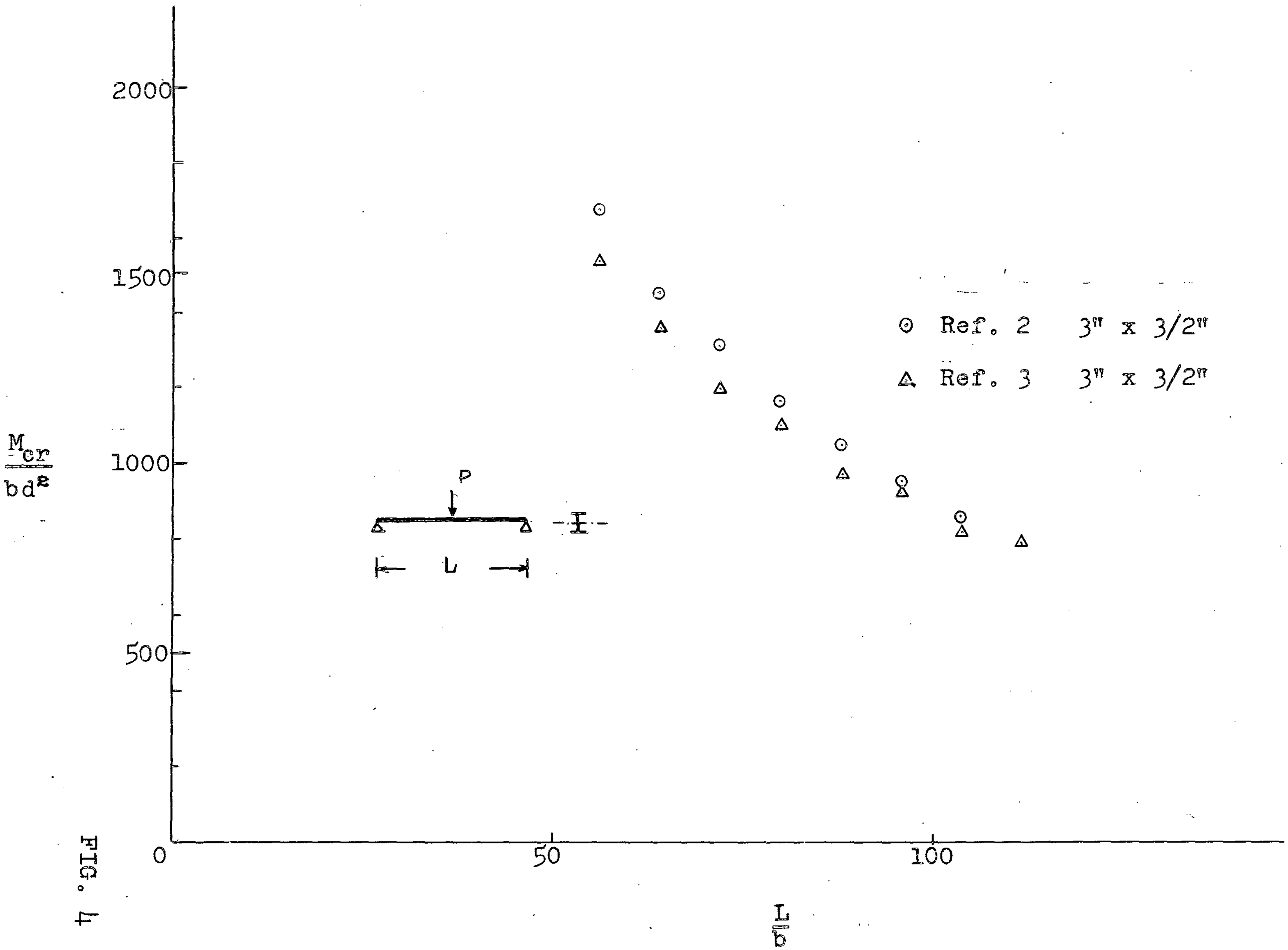
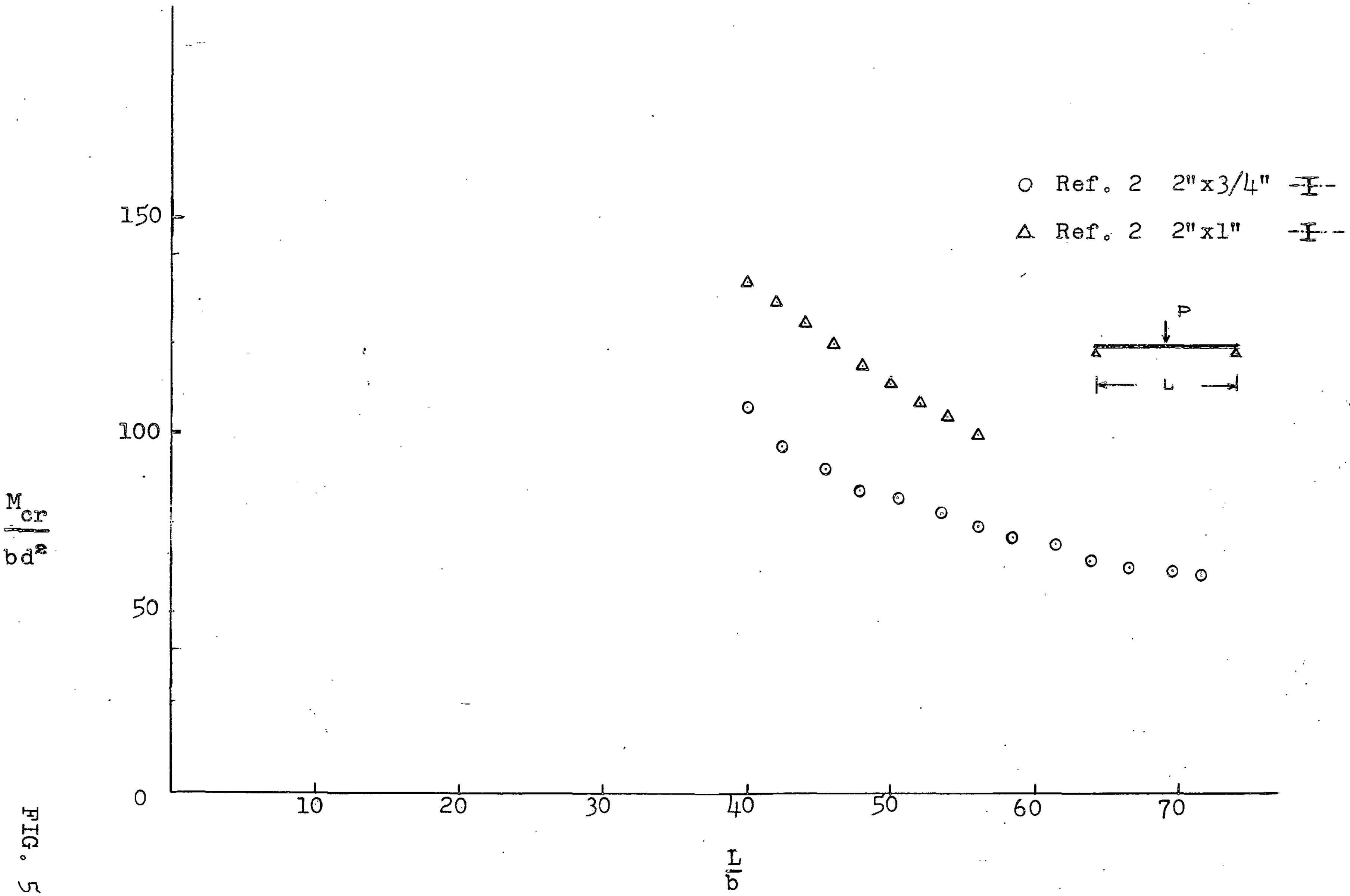

1.0
0.5
0
1000
2000
$\frac{M_{max.}}{M_p}$
$\frac{Ld}{bt}$
L
M
P
Ref. 1; σ_y = 60.55 ksi
Ref. 1; σ_y = 60.55 ksi
Ref. 2; σ_y = 20 ksi*
*Assumed

FIG. 3



2000
1500
1000
500
0
50
100
$\frac{M_{cr}}{bd^2}$
$\frac{L}{b}$
P
L
⊙ Ref. 2 3" x 3/2"
△ Ref. 3 3" x 3/2"

FIG. 4



○ Ref. 2 2"x3/4"
△ Ref. 2 2"x1"
P
L
$\frac{M_{cr}}{bd^2}$
150
100
50
0
10
20
30
40
50
60
70
$\frac{L}{b}$

FIG. 5

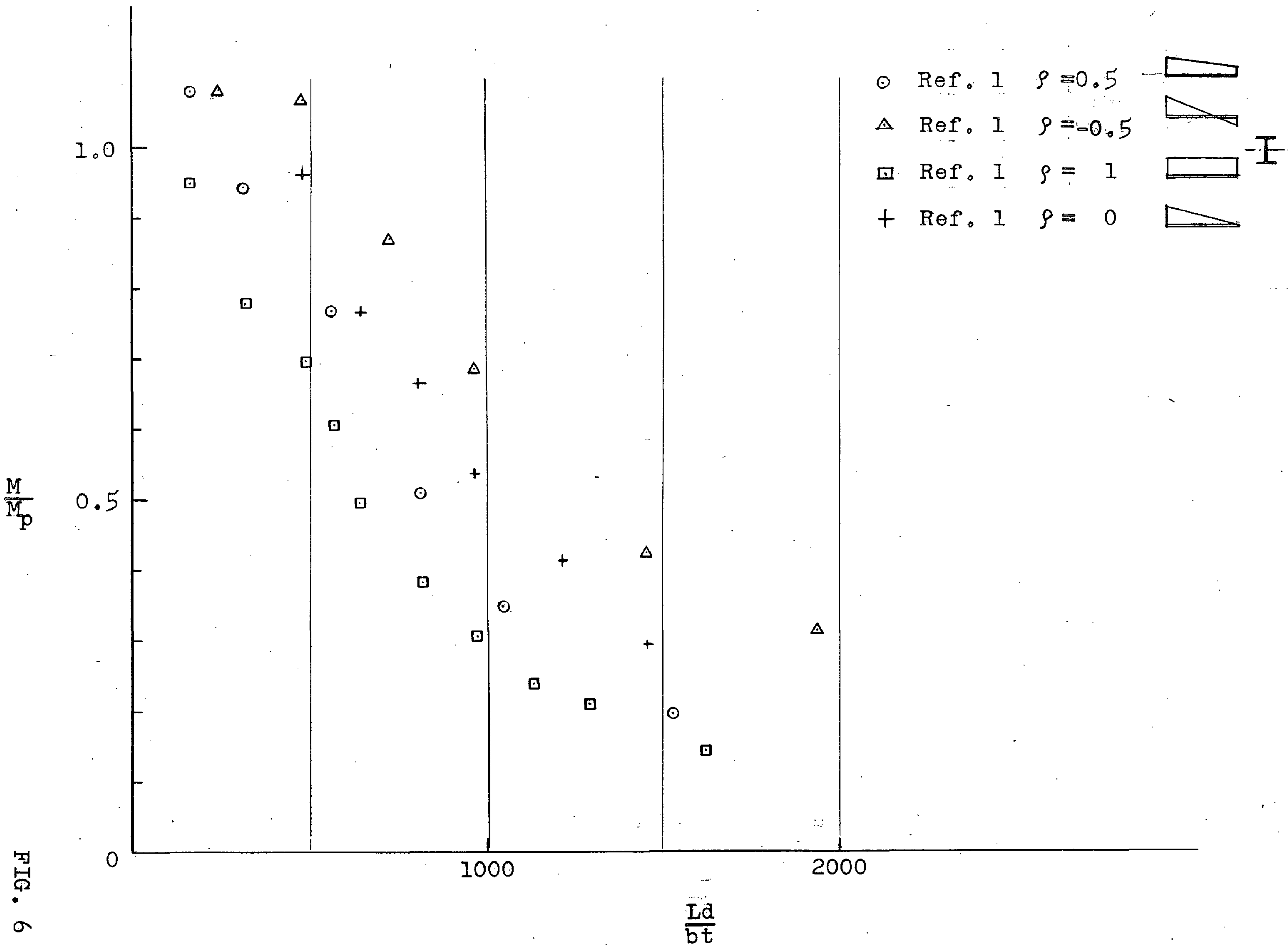


FIG. 6

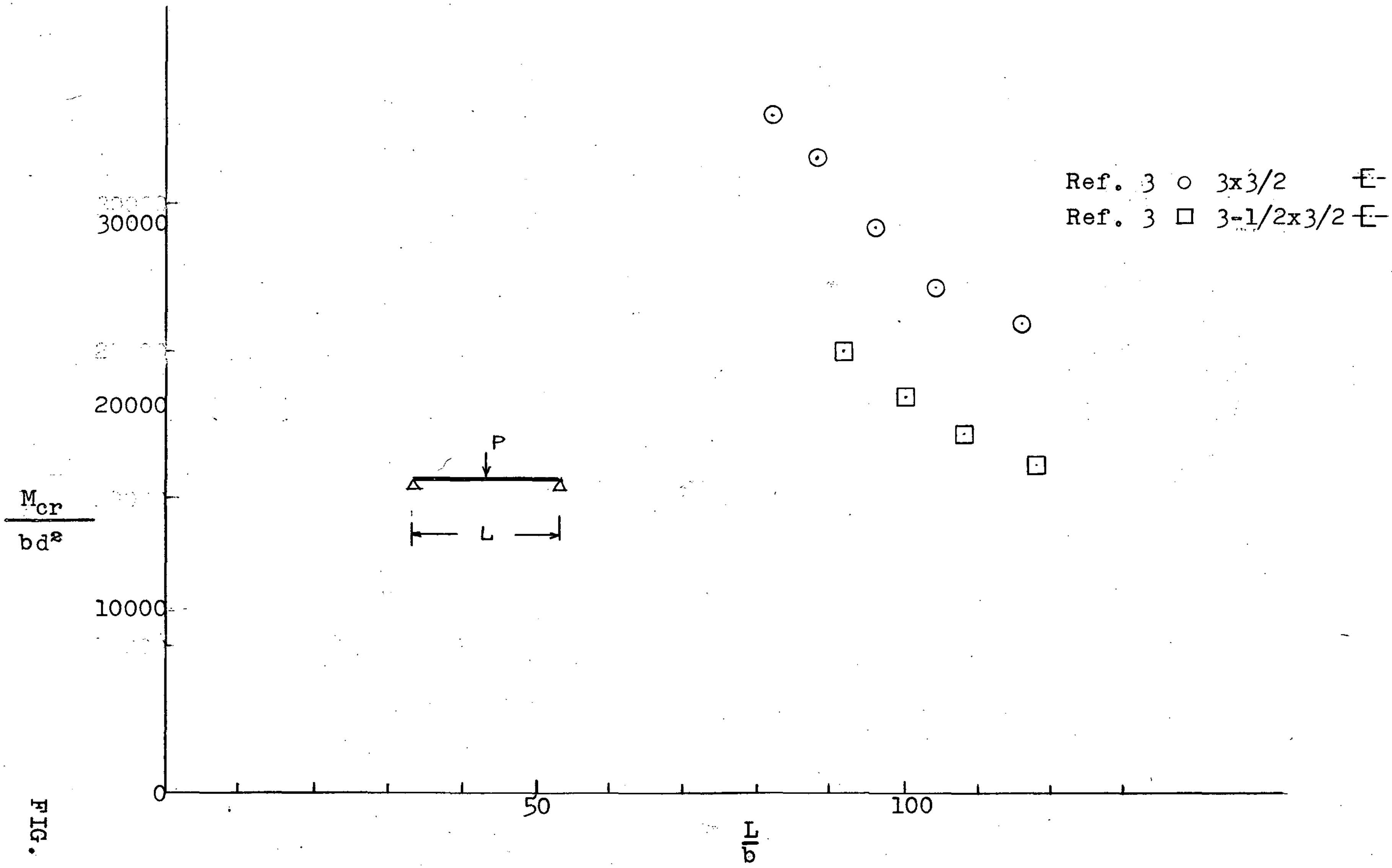

Ref. 3 ○ 3x3/2
Ref. 3 □ 3-1/2x3/2
30000
20000
10000
0
50
100
$\frac{M_{cr}}{bd^2}$
$\frac{L}{b}$
P
L

FIG. 7

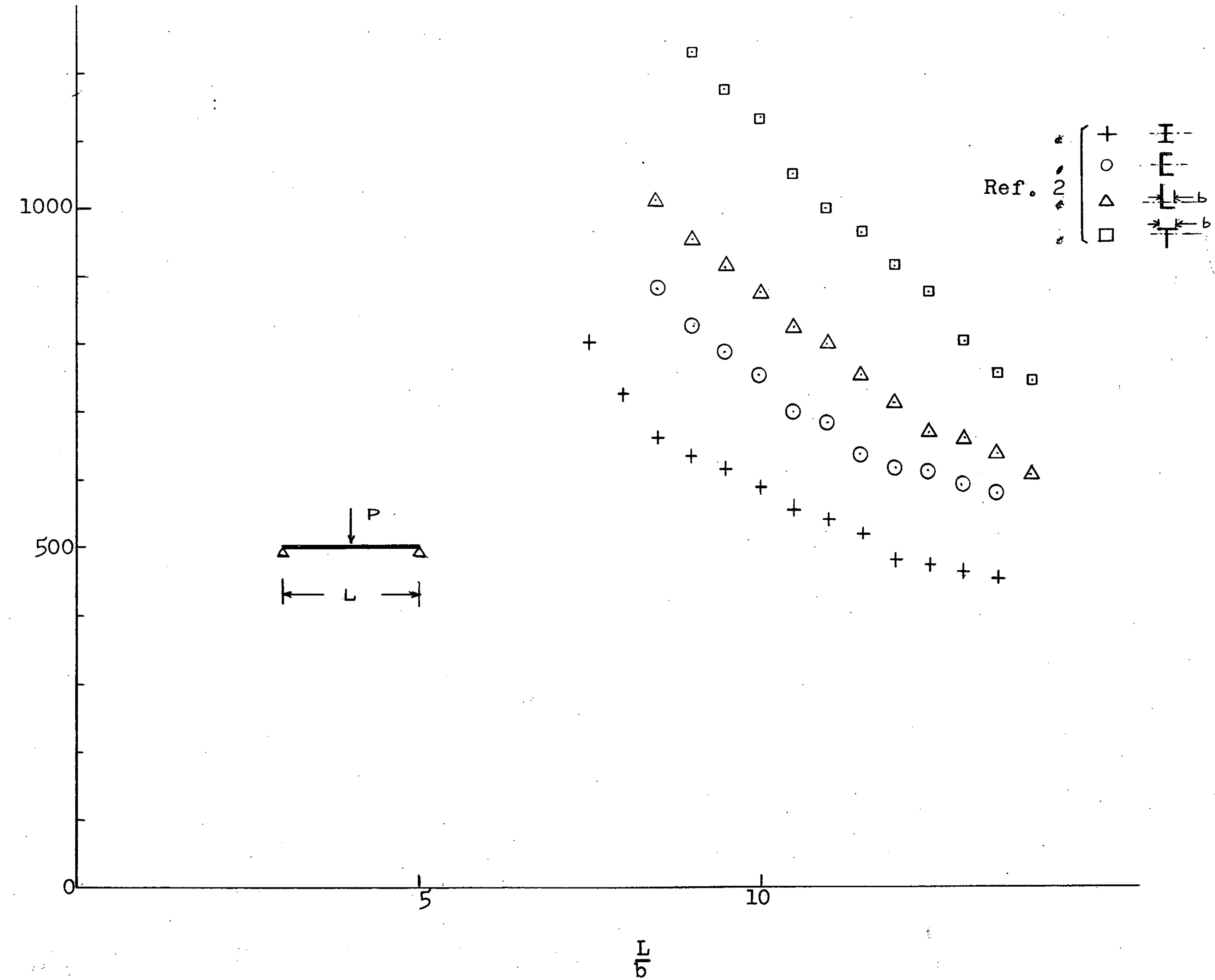


FIG. 8

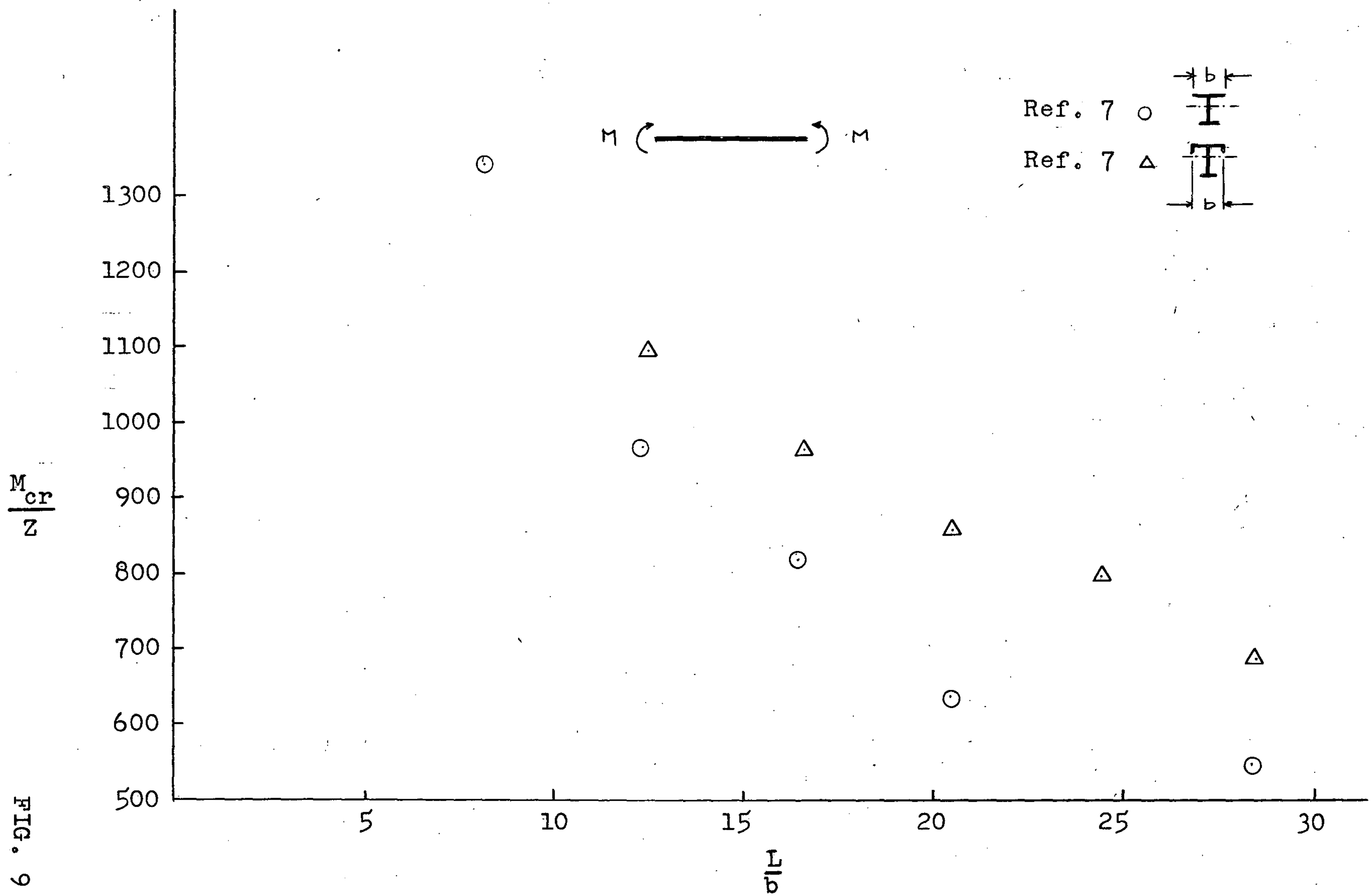

M
M
Ref. 7
Ref. 7
b
b
1300
1200
1100
1000
900
800
700
600
500
$\frac{M_{cr}}{Z}$
5
10
15
20
25
30
$\frac{L}{b}$

FIG. 9

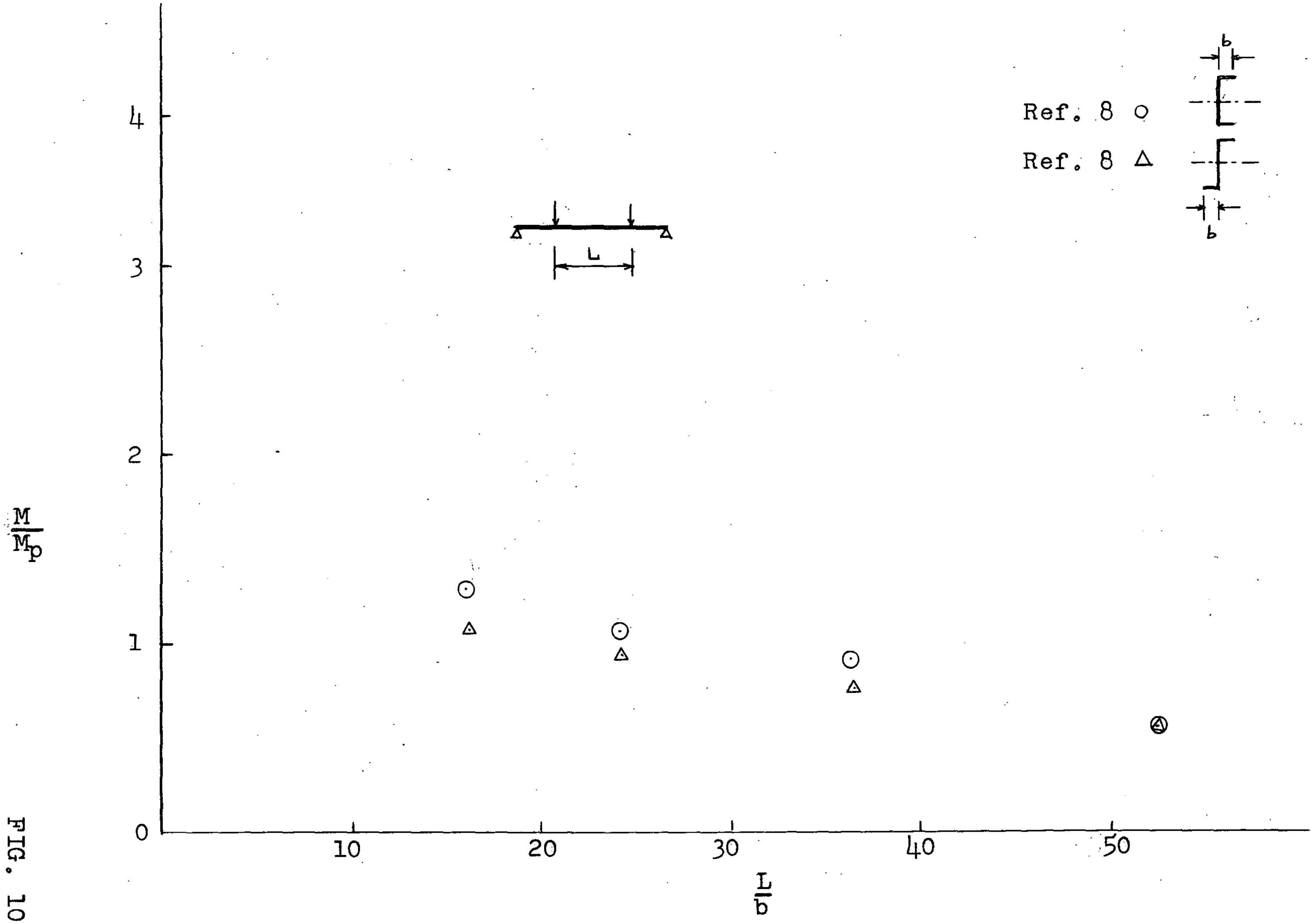


FIG. 10

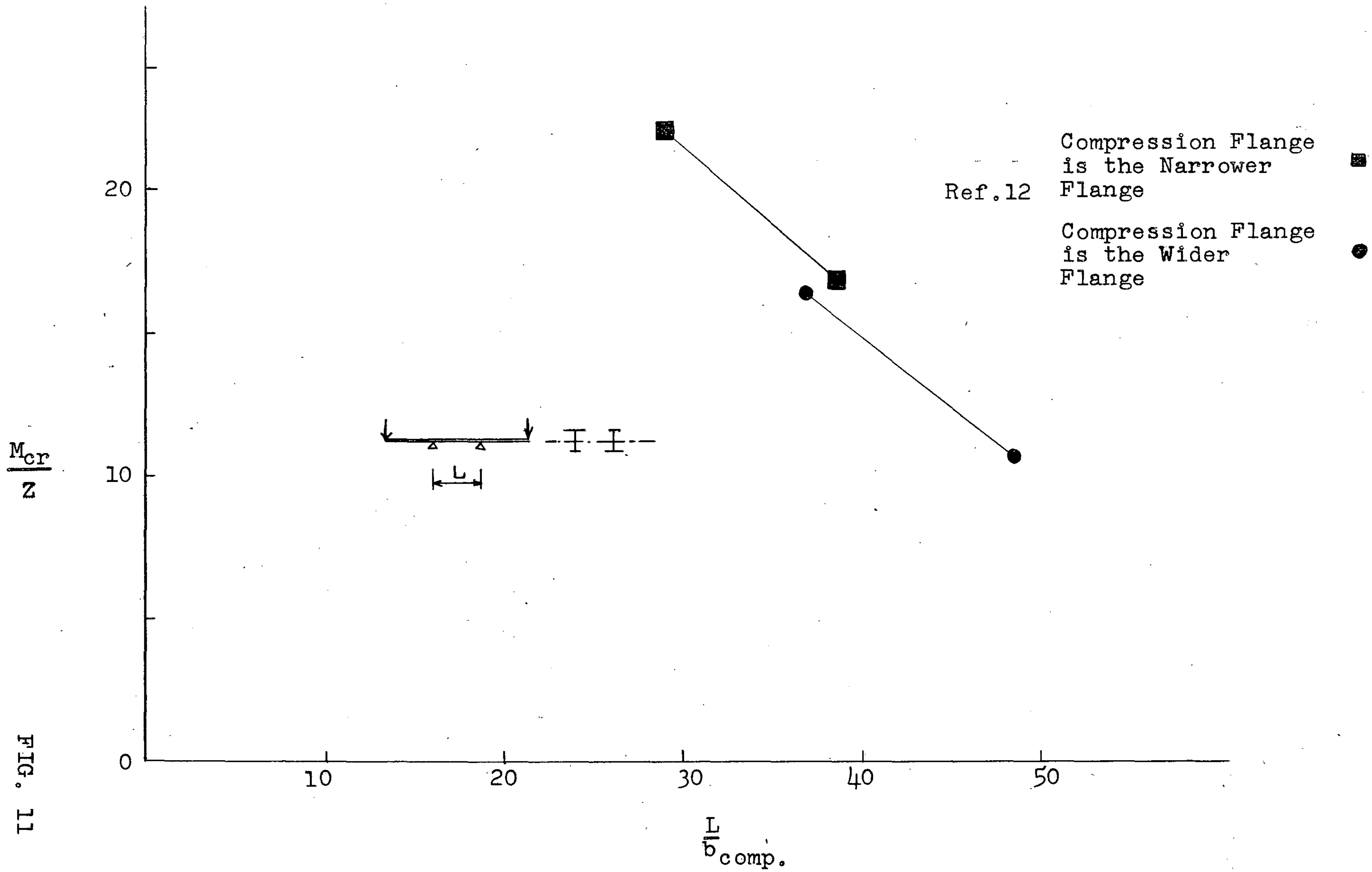
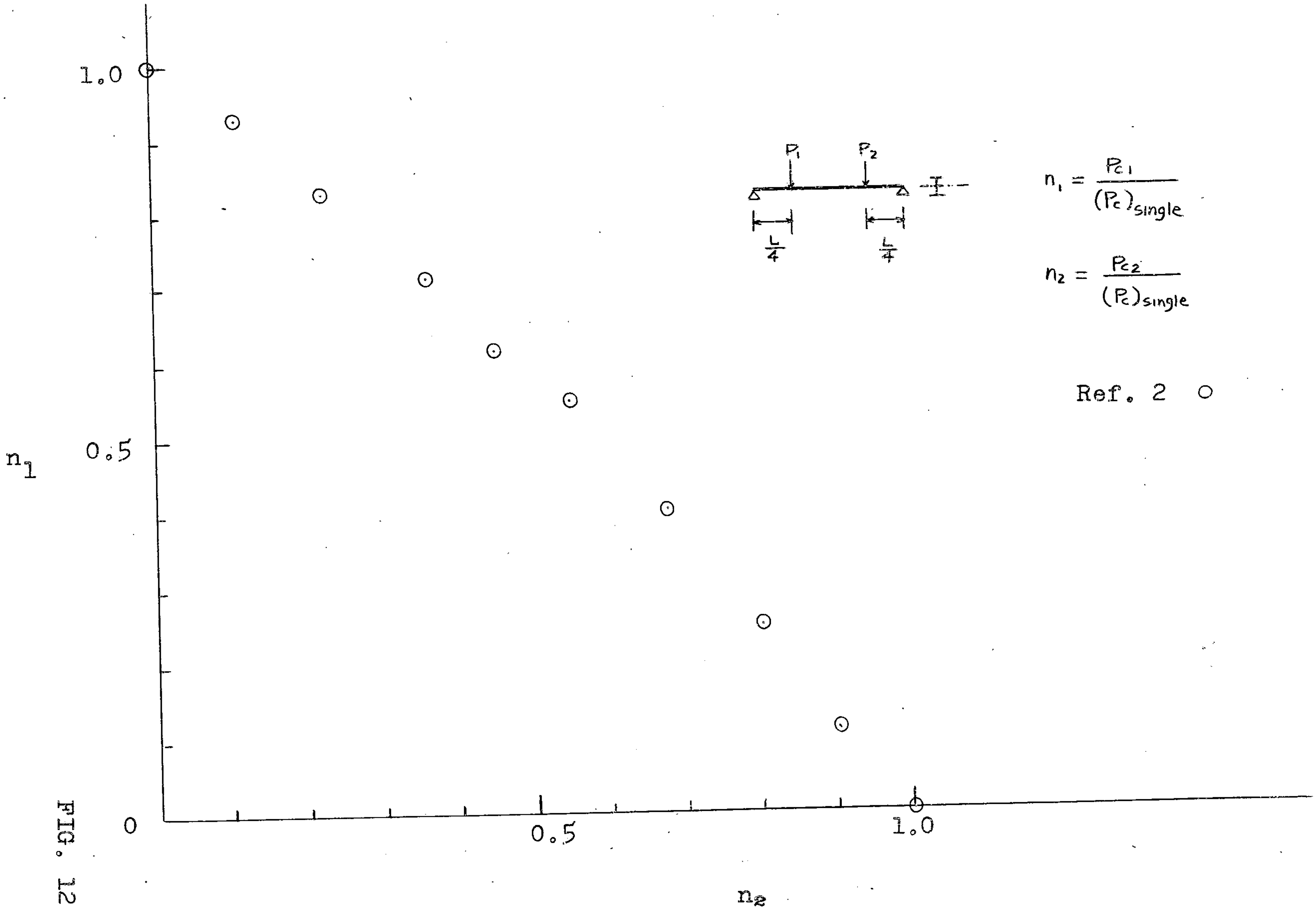

Compression Flange is the Narrower Flange
Ref.12
Compression Flange is the Wider Flange
Mcr / Z
L / b comp.
20
10
0
10
20
30
40
50
L

FIG. 11



P_1
P_2
$\frac{L}{4}$
$\frac{L}{4}$
$n_1 = \frac{P_{c1}}{(P_c)_{single}}$
$n_2 = \frac{P_{c2}}{(P_c)_{single}}$
Ref. 2
1.0
0.5
0
n_1
0.5
1.0
n_2

FIG. 12

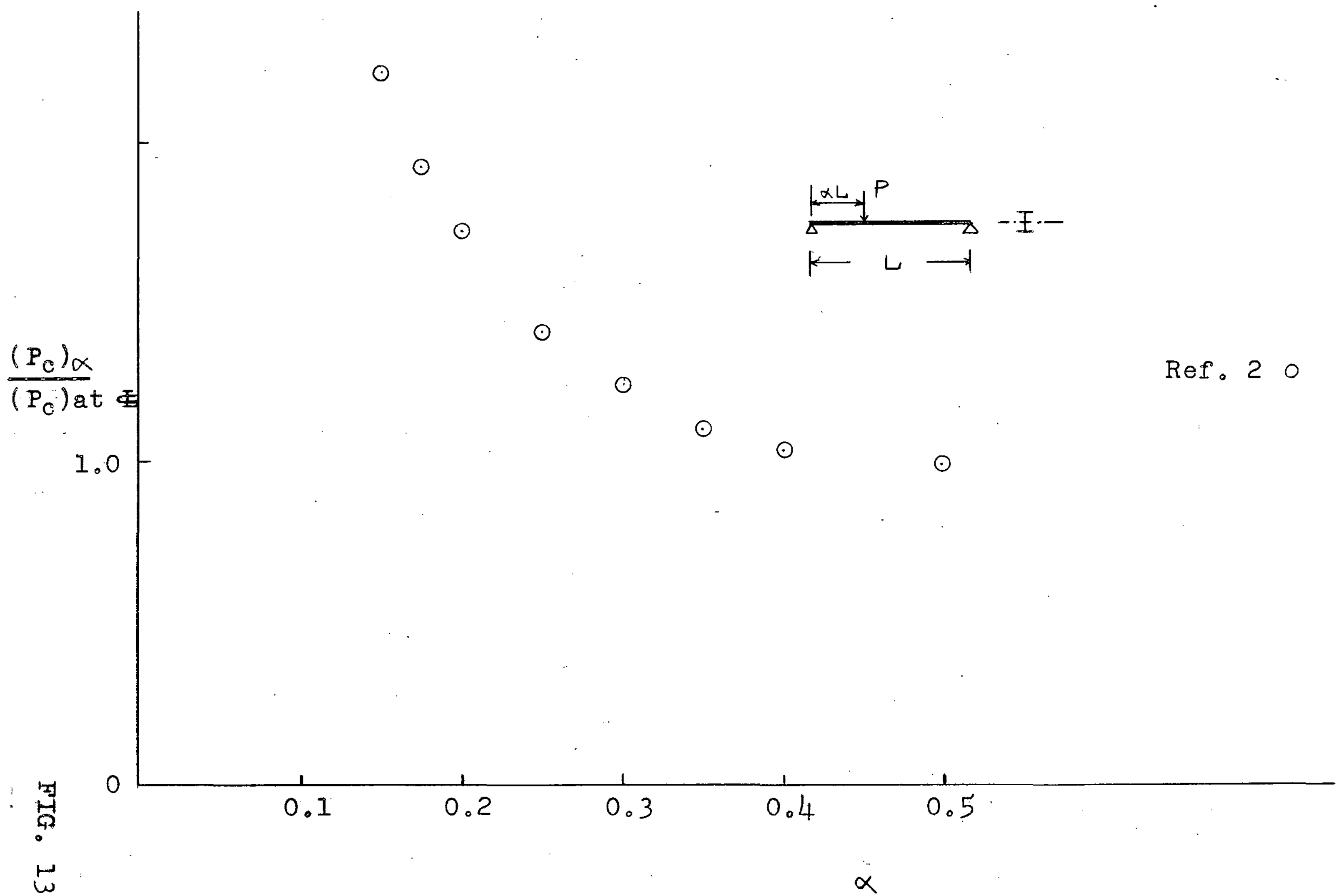


FIG. 13

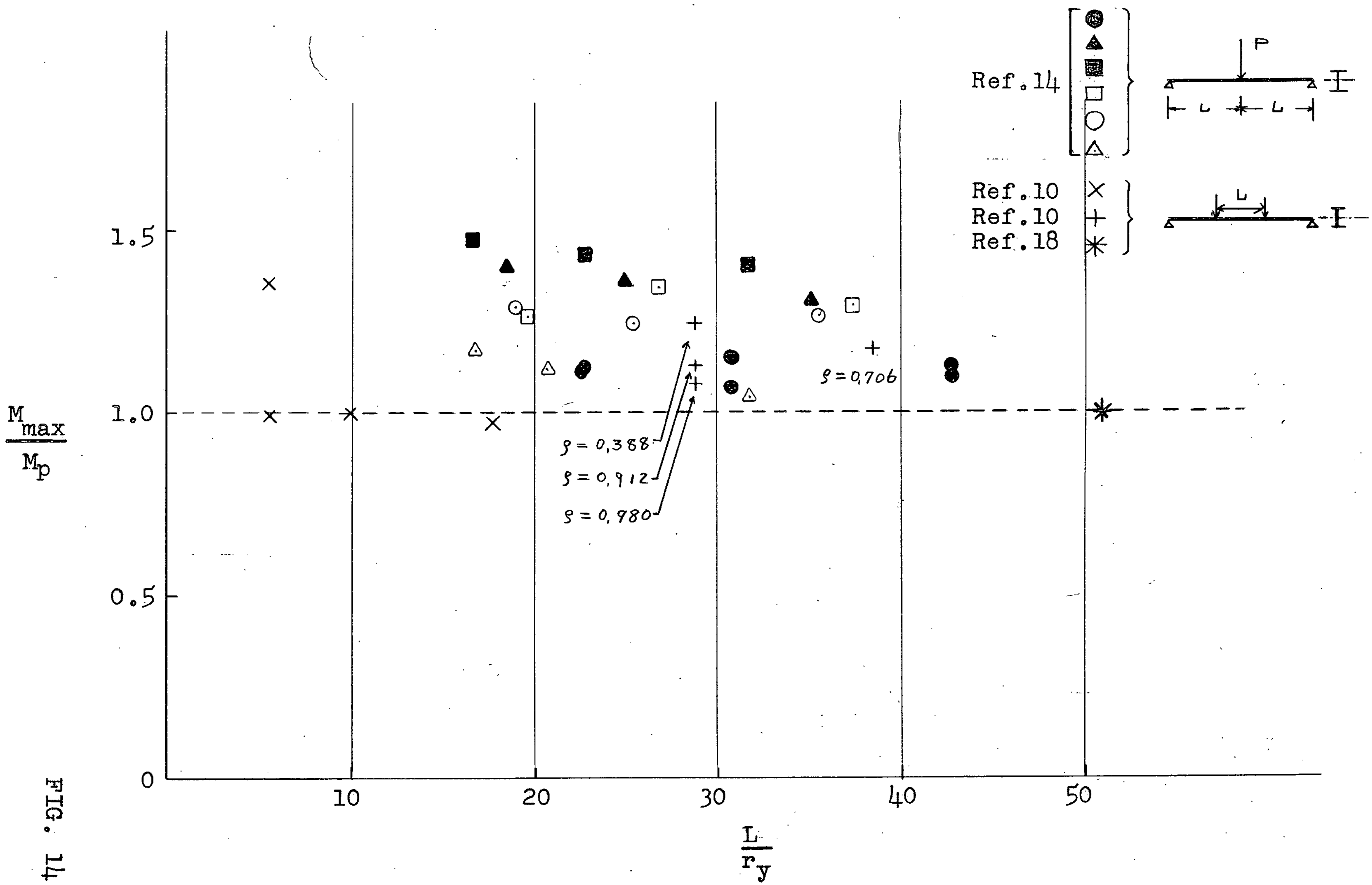

Ref. 14
P
L
L
Ref. 10
Ref. 10
Ref. 18
L
1.5
1.0
0.5
0
10
20
30
40
50
$\frac{M_{max}}{M_p}$
$\frac{L}{r_y}$
ρ = 0,388
ρ = 0,912
ρ = 0,980
ρ = 0,706

FIG. 14

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